Title: Pythagorean Modernized!

Brief Overview:

The TI-92 will be used to demonstrate the Pythagorean Theorem.

Link to Standards:

- **Problem Solving** Students will determine if a geometric figure is a right triangle given three numbers.
- **Communications** Students will discuss the geometric concept of squares of sides of a right triangle. Students will keep a written journal of these activities.
- **Reasoning** Students will make conjectures based on observed data.
- **Connections** Students will make connections between algebra and geometry.
- **Measurement** Students will use measurement of geometric shapes to obtain data.
- **Algebra** Students will investigate data to reasonable significant number of digits and recognize the use of significant digits.
- **Geometry** Students will use geometric properties of figures to write equations.

Grade/Level:

Grades 8-12

Duration/Length:

This activity will take one day with an additional 1 to 2 days should the teacher elect to use one or more of the extensions.

Prerequisite Knowledge:

Students should have working knowledge of the following:

- How to estimating and rounding
- How to construct geometric figures
- How to compute the square of a number
- Properties of geometric figures.

Objectives:

Students will:

- work cooperatively in groups.
- collect and organize data from resources during demonstrations.
- use numbers in a variety of equivalent forms.
- evaluate a situation and give appropriate support for their answer.
- use observed data to verify that a figure is a right triangle.
- observe the use of a macro.
- use a calculator to square numbers with several decimal places.

Materials/Resources/Printed Materials:

- TI-92 with view screen
- Pencils
- Paper
- Calculator
- Student Worksheets 1 - 2
- Teacher Notes

Development/Procedures:

- Verify the Theorem of Pythagoras given the values of a, b, and c.
- Observe a demonstration of the use of the TI-92 using a right triangle and the square macro.
- Record observations when the given right triangle is dragged to a new right triangle position.
- Record observations when the given right triangle is dragged so that the original right angle is less than and greater than 90 degrees.
- Communicate the above observations in the form of a written report.

Evaluation:

A written report based on journal entries and worksheets.

Extension/Follow Up:

1. Use an equilateral triangle macro in place of the square macro (see Note below) on the right triangle, evaluate observations, and record.

2. Use a circle macro in place of the square macro on the right triangle, evaluate observations, and record.

3. Compare and contrast the original activity with one or more of the above extensions and write a report.

4. Determine from the lengths of the sides of a triangle if the figure is an acute, a right, or an obtuse triangle.

Note: A macro is a sequence on interdependent constructions based on a defined initial object or objects and is used to construct one or more final objects.

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PYTHAGOREAN MODERNIZED
Teacher Notes

TI-92 Setup Instructions

Construct a RIGHT TRIANGLE using perpendiculars and acute angle vertices on segments. Construct **a square on each side of the triangle**. Label with the right angle marked and the areas of the squares in each square.

TI-92 General Notes: Press (green diamond) Z to undo the last step if necessary.
Move the cursor to highlight a menu selection and press enter OR type the number.

ON APPS 8:Geometry 3:new

down **cursor** to Variable: **rttriwsq enter enter**
(Rttriwsq is a suggested variable name for the figure of a right triangle with squares.)
Move the **cursor** to the right of center and down slightly.
F2 4:Line enter left **cursor enter** (to create a horizontal line)
F2 5:Segment enter left cursor for about 2 cm (on this line) **enter** (Figure 1)**.**

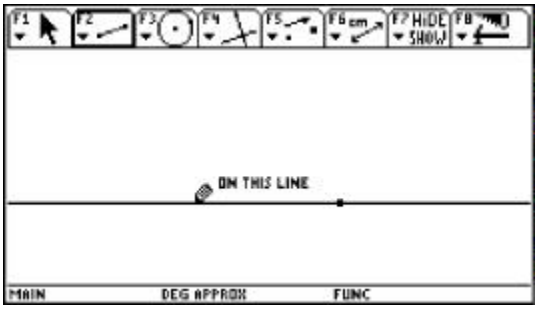


Figure 1

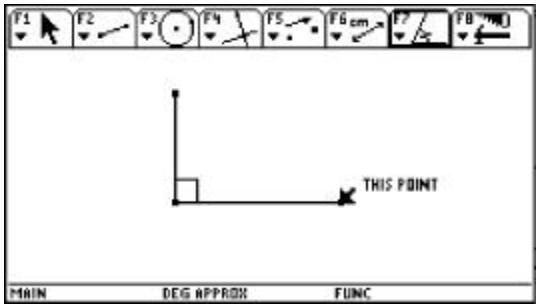


Figure 2

F7 1:Hide/Show right **cursor** to **(**this line**) enter** (leaves a horizontal segment).
Right **cursor** to the point **enter** .
F4 1:Perpendicular Line cursor to the left endpoint (thru this point) **enter**.
(Perpendicular to this segment) **enter.**
F2 5:Segment (this point) **enter** up **cursor** about 2 cm (on this line) **enter**.
F7 1:Hide/Show up **cursor** (this line) **enter** (hides vertical line). **F7 7:Mark Angle** down **cursor** to (this point) **enter**.
Down **cursor** to the vertex (this point) **enter**. Right **cursor** to endpoint on the right (this point) **enter**.
(A right angle marker should appear as shown in Figure 2.)

F3 3:Triangle Cursor to the point at the lower right (this vertex point) **enter**
Left **cursor** to (thru this vertex - of the right angle) **enter**
Up **cursor** to (thru this vertex - top) **enter**
The triangle has to be defined with sides in the reverse direction in relation to the square macro so that the square drawn on each side of the triangle will be exterior to the triangle. Figure 3. Press **enter** to complete the triangle.

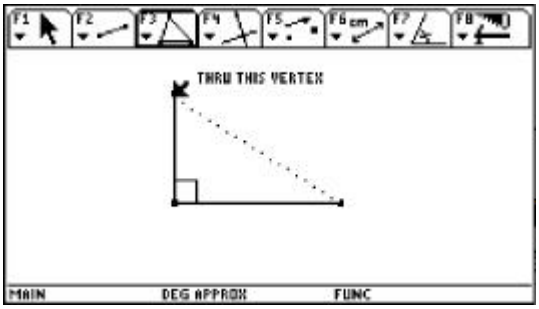


Figure 3

Now use the Macro "square1" to construct a square on each side of the triangle. **F4 6 1:Execute Macro** will show a list of macros available. Select by pressing its number. An alternate method is to select **F8 1:Open** right **cursor 2:Macro down** to Variable: **square1 enter** (Back to the triangle) **cursor** to (this triangle - on a side) **F4 6:Macro Construction** (or right **cursor) 1:Execute Macro 1: square1** (this side of the triangle) **enter**. Figures 4 - 6

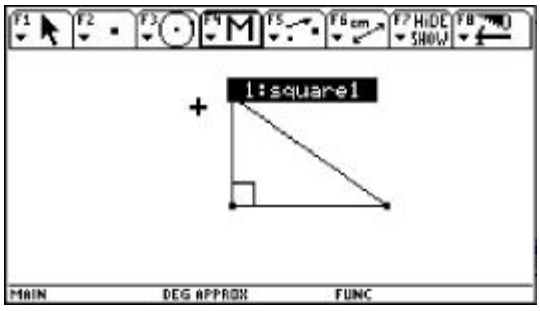


Figure 4

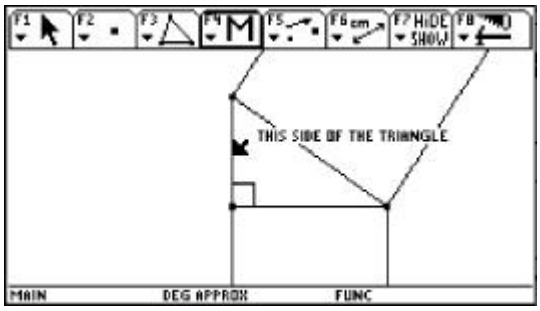


Figure 5

Figure 6

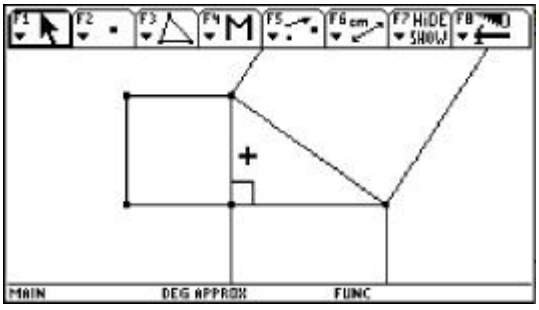
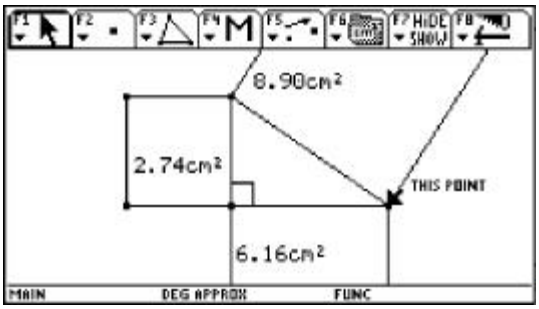


Figure 7



Move the **cursor** to the next side of the triangle and press **enter.** Repeat for the third side. **F6 2:Area** Move the **cursor** to the left side of a square to locate the area measurement inside the square. **enter**. Repeat for each square. It may be helpful to use the pointer **(F1 1: Pointer)** and the **hand** to drag the number in the left square to a better position. Figure 7.

The next step is optional. **F8 2:Save Copy as...** down **cursor** to Variable: and type in a variable name of your choice **enter enter** (to save the finished figure under a separate name). (This is a safe copy in case the other is inadvertently altered.)

PYTHAGOREAN MODERNIZED!
TEACHER PRESENTATION NOTES

Student Worksheet #1 is the warm up activity (approximately 5 minutes) for students to recall and check the algebraic application of the Pythagorean Theorem. Students should complete the worksheet and check and discuss the results briefly with his/her small group.

On the TI-92 view screen, bring up the prepared figure (rttriwsq) of a right triangle with squares constructed on each side. **ON APPS 8 2:Open** down **cursor** to Variable: right down **cursor** to "rttriwsq" **enter enter**.

Students should write in a journal during the presentation. Observations, sketches, data, ideas and questions are some types of notations that might be helpful.

Question: What do you notice about the numbers in each square? (Square measure, relation to each other, etc.) Allow sufficient time for reflection. Discuss some responses.

With the **pointer** selected, depress the **hand** . Any points blinking may be used to drag the triangle. Move the vertex and repeat the above question. **F1 1:Pointer** (this point - lower right) **enter Hand** (hold) **right cursor** until numbers change. Repeat several times. Move the upper left vertex up or down also to compare. Note the sum of the areas of the two smaller squares. If they do not add up exactly to the area of the larger square, why not? Use estimation, significant digits, rounding. More digits can be shown on the screen by selecting "this number" and then **+.**

Now use animation to collect additional data into tables (a data matrix).
F8 B:Data View [**2nd APPS F1 8:Clear Editor enter 2nd APPS** if data is already in the table]. Figures 8 and 9.

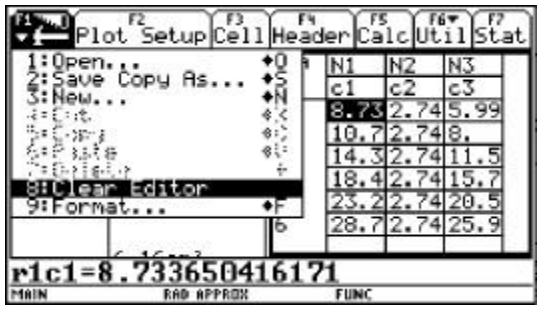


Figure 8

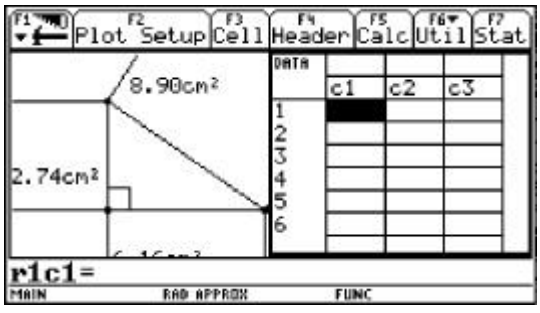
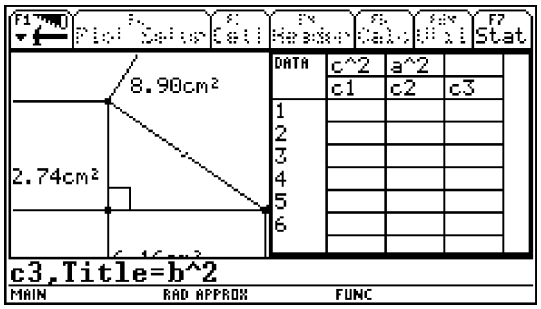


Figure 9

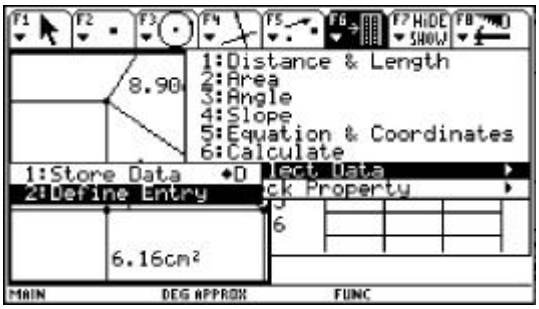
With the data/matrix editor active (**2nd APPS** to shift left/right), cursor to the column header (N1). Ask for suggestions for column headings. What data should be collected? In what order? Refer to worksheet #1 with $c^2 = a^2 + b^2$. Enter column headings. With the title box highlighted, **enter** . The cursor will be seen blinking at the bottom of the view screen. Type c^2, then **enter** . The title appears in column 1. Repeat using a^2 for the second column and b^2 for the third. Refer to Figure 10.

Figure 10



Return to the Geometry side (**2nd APPS**).

F6 7:Collect Data 2:Define Entry. (Figure 11). Move the cursor to (this number) **enter**. Starting with the largest, then the bottom and the left. (Figure 12).



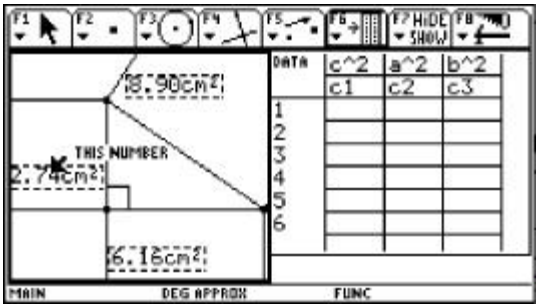
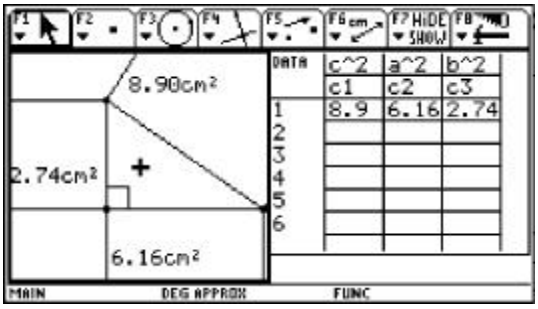


Figure 11 **Figure 12**

F6 7 1: Store Data enter The first three data values go into the tables. Figure 13.



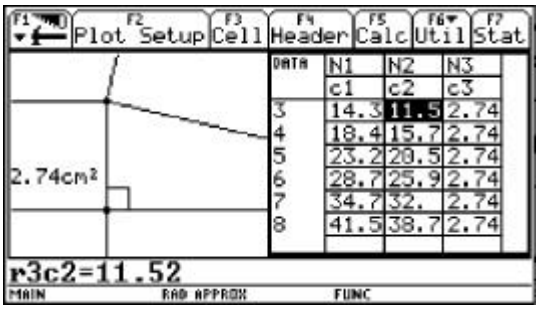


Figure 13 **Figure 14**

F7 3:Animate enter Select the lower right vertex. **Grab** with the hand **(hold**) and **Cursor to the left** (to move the point to the right!). Let go and watch the data fill the table. Figure 14. Use **enter** to pause the animation and then press the **ON** key to stop the animation when there is enough data, or the triangles are becoming too large.

The column headings may revert to N1, N2, and N3. If this occurs simply reenter the desired headings. Have students reflect on the data collected and note observations. Discuss observations, consider questions. Look for patterns, constants, variables, relationships, discrepancies. Note: Use **2nd APPS** to activate the data/matrix side of the screen and highlight a cell and observe the number of decimal places shown on the entry line at the bottom of the screen. Use **Mode F2 Split Screen cursor right 1:Full enter enter APPS 6: Data/Matrix Editor** to view all of the data collected. If you wish to view more of the number **F 1 9:Format cursor right** then **cursor up/down** to desired value. **enter.**

Since this triangle was constructed to have a right angle, all of the variations maintained the right angle. Next look at a triangle and the squares of the sides that may or may not be a right triangle.

2nd APPS to reactivate the Geometry side.
F8 C:Clear Data View enter
F8 3:New.... enter down to Variable: type **"trisqr2" enter enter.** Construct a right triangle using **F3 3:Triangle** by **down cursor enter right cursor enter.** Figure 15.

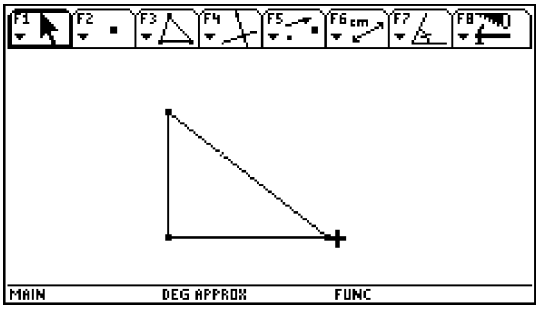


Figure 15

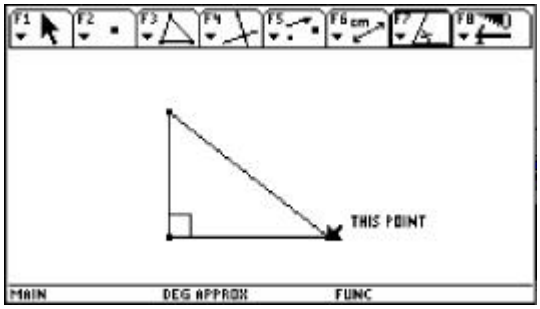


Figure 16

F 7 7:Mark Angle . Figure 16. Since this triangle was constructed without using perpendiculars it can be modified to any triangle. Execute **square1** macro. **F4 6:1 Execute Macro** place squares on each side of the triangle. Determine the area of each square using **F 6 2:Area**. Figure 17.

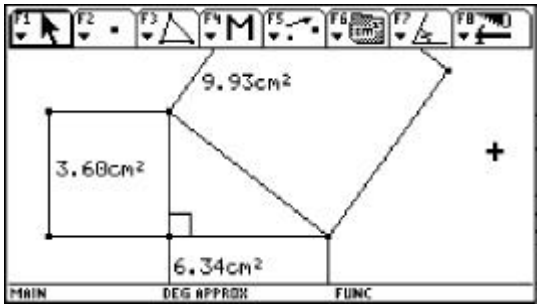


Figure 17

Use the hand to **grab and drag** a vertex. Note what happens to the shapes and values and values. Try several acute triangles. Note the numbers and any relationship. Try several obtuse triangles. Again note some numbers and any relationship. Discuss observations, questions, clarifications.

Look at a triangle that is "close" to a right triangle. Is it obtuse or acute? Why? Write inequalities for obtuse and acute triangles. Now complete worksheet #2 and summarize your findings in your journal.

Thought question: What if you had semicircles on each side of the triangle? Equilateral triangles?

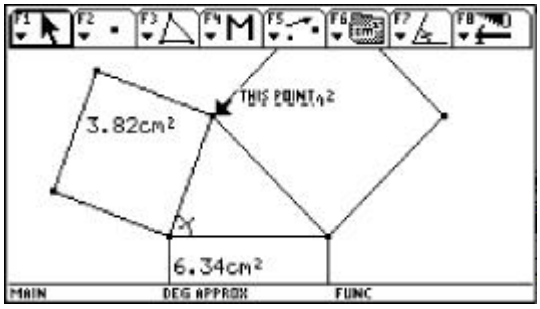


Figure 18

Create a "Square" Macro

The following steps may be used to write a macro for constructing a square on a segment.

Select the geometry application and a new figure, **Apps 8 3. Down cursor** to Variable and type desired name such as **"temp"** and **enter enter** . Move the cursor to center right of screen and press **F2 5 enter enter.** This determines the right endpoint of the segment. **Cursor left** horizontally about 2 cm and **enter.** This establishes the segment. (Figure 1.**)**

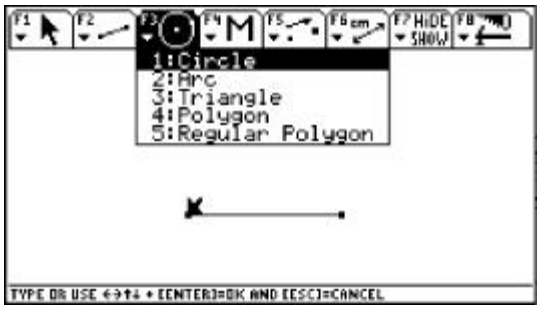


Figure 1

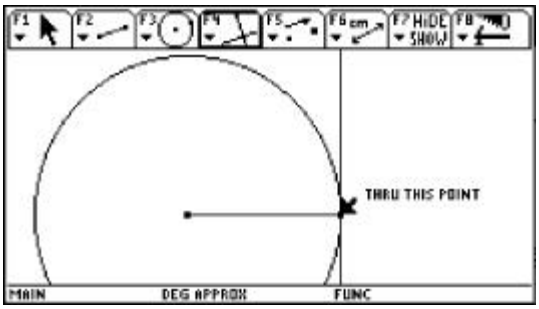


Figure 2

F3 1 enter. **Cursor right** along the segment until the circle intersects the right endpoint. **"This radius point"** will appear. **enter.** Press **F4 1 "Thru this point"** appears. **enter. "Perpendicular to this segment"** will appear. **enter. (**Figure 2**.)** Move the cursor to the left endpoint of the segment. **"Thru this point"** will appear**. enter. "Perpendicular to this segment"** will appear. **enter.** (Figure 3.)

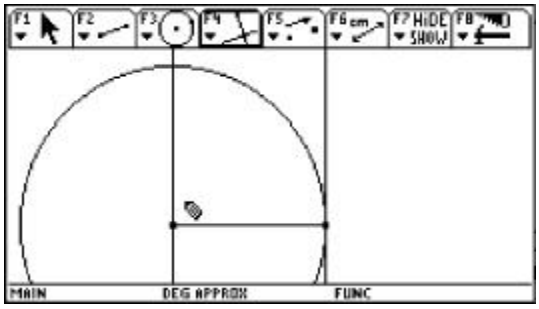


Figure 3

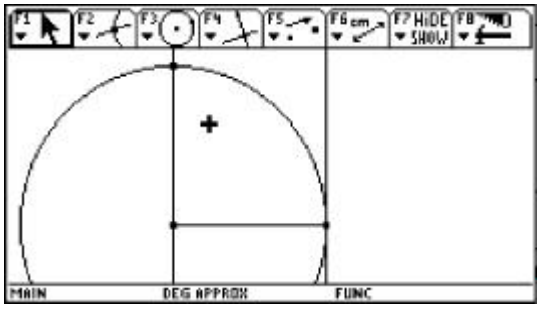


Figure 4

Move the cursor along the last line to a position below the intersection of the line and the circle. Press **F2 3 enter. "This line"** will appear and the line will be dashed. Move the cursor to the circle. **"This circle"** will appear. **enter.** A point will appear at that intersection. (Figure 4.) Press **F4 1.** Move the cursor to the line through the center of the circle. **"Perpendicular to this line"** appears. **enter.** Move the cursor to the point of intersection. **"Thru this point"** will appear. **enter**. Place the cursor on the last perpendicular line drawn. Press **F2 3. enter.** Move the cursor to the tangent line at this intersection and **enter.** A point appears at this intersection.

The next series of steps will establish the square drawn as a closed figure.

Position the cursor at one of the vertices of the square. Press **F3 4. "This point"** will appear. **enter.** Move the cursor to each of the vertices in order clockwise and press **enter** when **"This point"** appears. (Figure 5.) The cursor should be back at the starting point.

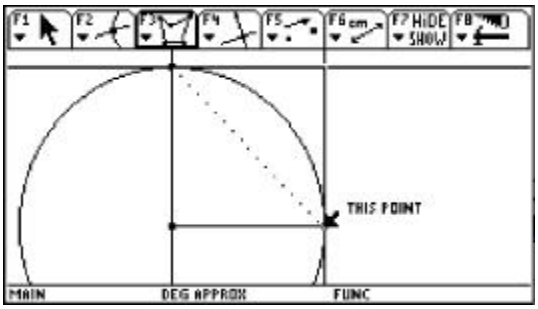


Figure 5

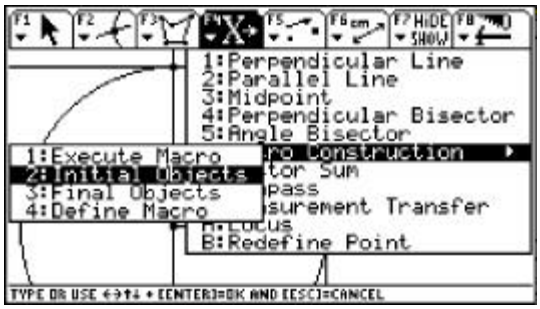


Figure 6

The next series of steps will create the macro from this figure.

Press **F4 6 2.** (Figure 6.) Move the cursor to the initial segment. **"Which object"** will appear. **enter enter.** The segment should be dashed. **F4 6 3** Move the cursor to one of the adjacent sides of the square. **"Which object?"** appears. **enter 2.** The square will be dashed. **F4 6 4.** Type in the name **square1. enter enter.** Cursor down to Object name and type **square1 enter enter.** Cursor down to Variable and type **square1. enter enter.**

The next series of steps will test the macro.

Press **F2 5** Move the cursor to a clear area of the screen and **enter.** Move the cursor horizontally a distance. **enter.** Press **F4 6 1** to select and execute the macro. If more than one macro name appears, select **square1** and **enter.** Move the cursor to the segment. **"This segment"** will appear. **enter.** (Figures 7 and 8.)

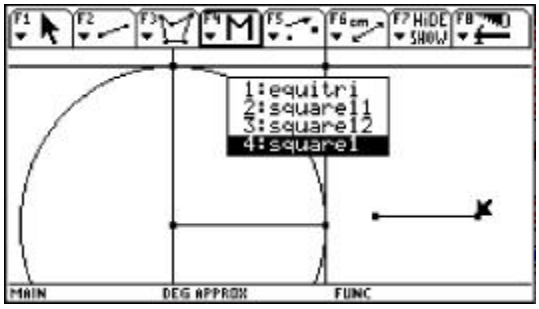


Figure 7

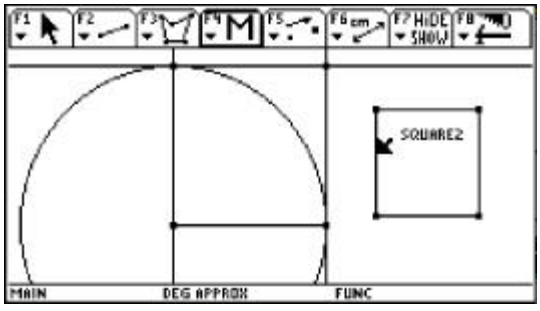
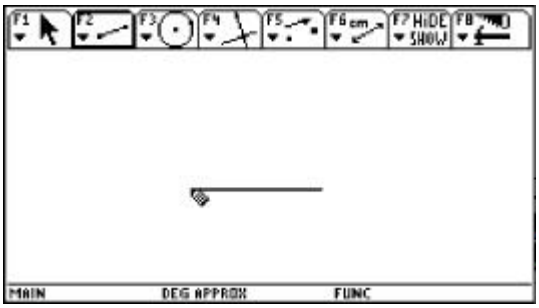


Figure 8

Equilateral Triangle Macro

The following steps may be used to write a macro for an equilateral triangle.

Select the geometry application and a new figure. **Apps 8 3. Down cursor** to variable and type desired variable name such as **"temp". enter enter.** Position the pointer center right on the screen and press **F2 5 enter enter.** This determines the right endpoint of the segment. **Cursor left** horizontally about 2 cm and **enter.** This establishes the left endpoint of the segment. (Figure 1)



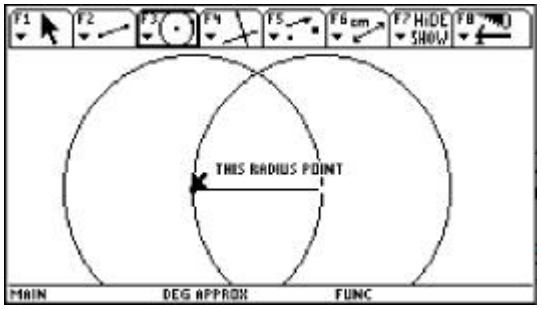
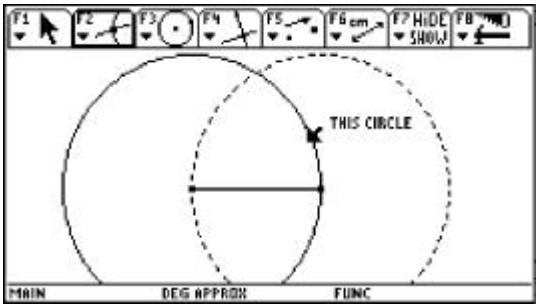


Figure 1 **Figure 2**

Press **F3 1. enter.** Cursor right along the segment until the circle intersects the right endpoint. **"This radius point"** appears. **enter enter.** The words **"Point at this intersection"** appears. **Cursor left** and expanding circle appears. Continue to the left until the second circle intersects with the left endpoint of the segment and the words **"This radius point"** appear. **enter.** (Figure 2). Select **F2 3 enter.** The pencil icon appears. Move the pencil to one of the circles until the words **"This circle"** appear. **enter.** The selected circle appears dashed. Move the cursor to the other circle until the words **"This circle"** appear. (Figure 3) and press **enter**. A point will appear at the intersection of the two circles above and below the segment if both sides of the segment art visible**.**



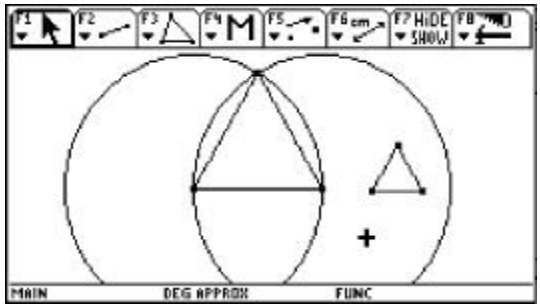


Figure 3 **Figure 4**

Press **F3 3** and construct the triangle. **"Thru this vertex"** will appear at each point connected. The figure is ready for the creation of the actual macro.

F4 6 2 Cursor to the original segment until the words **"Which object?"** appear **enter enter.** The segment should be dashed. **F4 6 3.** Move the cursor to the second side of the triangle until the words **" Which object?"** appear. **enter enter.** This segment should be dashed. Press **F4 6 4.** Type a name for the macro and the object. Press **enter enter.** Key in a variable name of your choice. **enter enter.**

Test the macro.

Draw a short segment in a clear area of the screen. **F4 6:1: Execute Macro.** The macro name(s) will appear on the screen. Select the appropriate macro if more than one appears and press **enter . "This segment"** appears. **enter.** The completed equilateral triangle appears and should be a different size. (Figure 4).

Pythagorean Modernized!

Student Worksheet 1 Name _____

Date _____

The Pythagorean Theorem: If **a** and **b** are the legs of a right triangle, and **c** is the hypotenuse, then $\mathbf{c^2 = a^2 + b^2}$.

1. Do the following sets of numbers satisfy the Pythagorean Theorem?

   a. 3, 4, 5 _____

   b. 5, 13, 12 _____

   c. 3.7, 1.2, 3.5 _____

   d. 1/2, 2/5, 5/29 _____

2. Given a = 8 in. and b = 15 in. Find c. c = _____

3. Two sides of a right triangle measure **9** m. and **40** m. What would be the measure of the third side? _____

   [Hint: There are *two* answers to this question!]

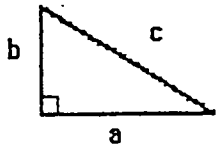
Pythagorean Modernized!

Student Worksheet 2

Name _____

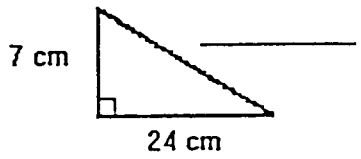
Date _____

The Pythagorean Theorem: If triangle ACB is a right triangle, then $c^2 = a^2 + b^2$.

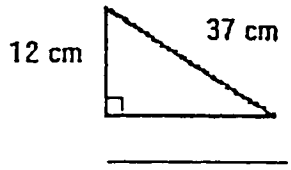


1. Find the missing sides.

a.

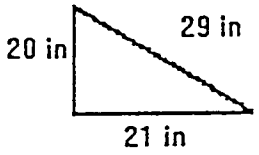


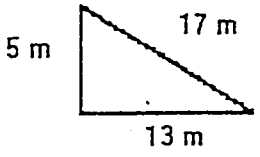
b.

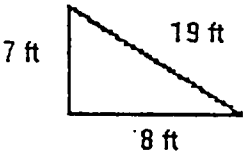


2. Are the following figures right triangles?

a. _____ b. _____ c. _____







3. Draw a square on each side of the triangle below and calculate the area of each of the squares. Add the areas of the two smaller squares. How does the sum compare to the area of the largest square? _____
Will this always be true? _____ Why/Why not? _____
_____

ANSWERS TO WORKSHEET PROBLEMS

Worksheet 1.

1. a. Yes

   b. Yes

   c. Yes

   d. No

2. c = 17 in.

3. Two answers: 41 m. or 1519 = 7 31 m.

Worksheet 2.

1. a. 25 cm

   b. 35 cm

2. a. Yes

   b. No

   c. No--not a triangle!

3. The areas are equal; No; It will only be true if the triangle is a right triangle